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Formulas for the Pressure and Bulk Modulus in Uniaxial Strain

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For an isotropic elastic solid, the pressure $p = p_u(\rho)$ in a state of uniaxial strain at density ρ generally differs from the pressure $p = p_h(\rho)$ in a state of hydrostatic stress at the same density. Several researchers have used pressure/shear (or oblique plate impact) tests to determine p_u and the corresponding uniaxial bulk modulus $K_u \equiv \rho dp_u/d\rho$. The pressure/shear tests yield uniaxial longitudinal and shear moduli, L_u and G_u, as functions of ρ. A common procedure is to integrate the approximate relation $K_u \approx L_u - 4/3\ G_u$ to obtain the pressure-density relation $p = p_u(\rho)$ in uniaxial strain. It is shown here that the integration of this approximate relation between the moduli can be avoided altogether by utilizing the exact formula $p_u = \sigma_1 - 2/3\ [(\rho/\rho_0)^2 - 1]\ G_u$, where σ_1 denotes the longitudinal stress (positive in compression). The bulk modulus K_u is computed exactly from this formula, and the error in approximating it by $L_u - 4/3\ G_u$ is determined.

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1 INTRODUCTION

We consider only isotropic elastic materials, and for simplicity thermal effects are neglected until §5. Under these conditions, the pressure is typically assumed to be a function of density only. However, nonlinear elasticity theory predicts that the pressure also depends on the shear strain, although isotropy implies this effect is necessarily of second order; cf. Scheidler [1]. In §4 we derive exact formulas for the pressure and bulk modulus in a state of uniaxial strain. The effect of shear strain can be seen by comparing these results with the corresponding relations for a state of hydrostatic stress (§3). Our results are based on exact formulas for the speeds of acceleration waves (§2). Applications to the analysis of data from pressure/shear tests are discussed in §5.

2 ACCELERATION WAVE SPEEDS

Let $\mathbf{F}$ denote the deformation gradient relative to the undeformed and unstressed state. The left Cauchy-Green tensor $\mathbf{B} \equiv \mathbf{F}\mathbf{F}^{\mathrm{T}}$ has principal values $b_i = \lambda_i^2$, where λ_i are the principal stretches, and

$$\det \mathbf{F} = \sqrt{b_1 b_2 b_3} = \lambda_1 \lambda_2 \lambda_3 = \frac{1}{\tilde{\rho}}, \quad \tilde{\rho} \equiv \frac{\rho}{\rho_0}, \tag{2.1}$$

where ρ and ρ_0 denote the densities in the deformed and undeformed state. The principal axes of $\mathbf{B}$ are the principal axes of strain in the deformed state. Since the material is isotropic and elastic, these axes are also the principal axes of the Cauchy stress tensor $\mathbf{T}$, and $\mathbf{T}$ is an isotropic function of $\mathbf{B}$. This implies that there is a single function $\hat{t}$ such that the principal stresses t_i are given by

$$t_i = \hat{t}(b_i, b_j, b_k) = \hat{t}(b_i, b_k, b_j), \tag{2.2}$$

for any permutation i, j, k of $1, 2, 3$; cf. Truesdell & Noll [2, §48]. It follows that the pressure

$$p \equiv -\tfrac{1}{3}\mathrm{tr}\,\mathbf{T} = -\tfrac{1}{3}(t_1 + t_2 + t_3) \tag{2.3}$$

is a symmetric function of b_1, b_2, b_3. Analogous results hold in terms of the principal stretches λ_i or in terms of various principal strain measures, e.g., $\lambda_i - 1$, $\frac{1}{2}(b_i - 1)$, $\frac{1}{2}(1 - 1/b_i)$, or $\ln \lambda_i$.

The speed U_i of a longitudinal acceleration wave propagating along the ith principal axis of strain is given by

$$\rho U_i^2 = 2b_i \frac{\partial t_i}{\partial b_i} = \lambda_i \frac{\partial t_i}{\partial \lambda_i} = \frac{\partial t_i}{\partial \ln \lambda_i}. \tag{2.4}$$

The speed U_{ij} of a transverse (or shear) acceleration wave propagating along the ith principal axis of strain with jump in acceleration parallel to the jth principal axis ($j \neq i$) is given by Ericksen's formula:

$$\begin{aligned} \rho U_{ij}^2 &= b_i \left(\frac{\partial t_i}{\partial b_i} - \frac{\partial t_i}{\partial b_j} \right), \quad \text{if} \quad b_i = b_j, \\ &= b_i \frac{t_i - t_j}{b_i - b_j}, \quad \text{if} \quad b_i \neq b_j. \end{aligned} \tag{2.5}$$

All quantities in (2.4) and (2.5) are evaluated at the wave front. These wave speeds are in the deformed material (i.e., Eulerian); the corresponding Lagrangian wave speeds are obtained by dividing by λ_i. Proofs of (2.4) and (2.5) can be found in Truesdell & Noll [2, §74] and Wang & Truesdell [3, §VI.5]. These formulae do not require that the region ahead of the wave be at rest or in a homogeneous state of strain. However, when these conditions are satisfied, (2.4) and (2.5) also apply to the speeds of plane infinitesimal sinusoidal waves; cf. Truesdell & Noll [2, §73].

3 HYDROSTATIC STRESS

For a purely dilatational deformation, we have

$$b_i = \tilde{\rho}^{-2/3} \quad \text{and} \quad t_i = -p \quad (i = 1, 2, 3). \tag{3.1}$$

In this hydrostatic stress state, every axis is a principal axis of stress and strain, and the pressure p is a function p_h of ρ or $\tilde{\rho}$. Here and below, an "h" subscript denotes the hydrostatic stress state. From (2.2), (2.4), (2.5)$_1$, and (3.1), it follows that for a given density ρ there is a single longitudinal wave speed $U_i = U_{L,h}$ and a single shear wave speed $U_{ij} = U_{S,h}$, and that

$$\frac{dp_h}{d\rho} = U_{L,h}^2 - \tfrac{4}{3} U_{S,h}^2 ; \tag{3.2}$$

cf. Wang & Truesdell [3, §VI.5]. A different proof of this well-known result is given by Truesdell & Noll [2, §75]. We assume p_h is a strictly increasing function of ρ. Then (3.2) implies the longitudinal wave speed is greater than the shear wave speed. With the longitudinal, shear, and bulk moduli defined by

$$L_h \equiv \rho U_{L,h}^2 \qquad G_h \equiv \rho U_{S,h}^2 \tag{3.3}$$

$$K_h \equiv \rho \frac{dp_h}{d\rho} = \tilde{\rho} \frac{dp_h}{d\tilde{\rho}} , \tag{3.4}$$

(3.2) implies the well-known relation

$$K_h = L_h - \tfrac{4}{3} G_h . \tag{3.5}$$

We use a zero subscript to denote functions evaluated at the undeformed and unstressed state where $\lambda_i = b_i = \tilde{\rho} = 1$; in particular, $K_0 = L_0 - \frac{4}{3}G_0$. By (3.4),

$$p_h/K_0 \approx (\tilde{\rho} - 1) + a_0(\tilde{\rho} - 1)^2 \,, \tag{3.6}$$

where the dimensionless constant a_0 is given by

$$a_0 = \frac{1}{2K_0}\,\frac{d^2 p_h}{d\tilde{\rho}^2}\bigg|_0 = \frac{1}{2}\left(\frac{dK_h}{dp_h}\bigg|_0 - 1\right) . \tag{3.7}$$

4 UNIAXIAL STRAIN

For a state of uniaxial strain along the 1-axis,

$$\lambda_1 = \sqrt{b_1} = 1/\tilde{\rho}, \quad \lambda_2 = \lambda_3 = b_2 = b_3 = 1 \,, \tag{4.1}$$

and (2.2) implies $t_2 = t_3$. The principal stresses t_i are positive in tension; if $\sigma_i \equiv -t_i$ then σ_i is positive in compression. We use a "u" subscript to denote uniaxial strain and consider only waves propagating along the 1-axis into a uniaxially strained material. The Eulerian wave speed $U_{L,u} = U_1$ of a longitudinal acceleration wave is given by (2.4) with $i = 1$, and by (4.1) we also have

$$L_u \equiv \rho\, U_{L,u}^2 = \tilde{\rho}\,\frac{d\sigma_1}{d\tilde{\rho}} = \rho\,\frac{d\sigma_1}{d\rho} \,. \tag{4.2}$$

It follows that a longitudinal acceleration wave can propagate only if $d\sigma_1/d\tilde{\rho} > 0$, i.e., if σ_1 is a strictly increasing function of $\tilde{\rho}$, which we now assume. By (4.1), the material is strained iff $\tilde{\rho} \neq 1$ iff $b_1 \neq b_2$. In this case $(2.5)_2$ and (4.1) imply the following formulas for the Eulerian speed $U_{S,u} = U_{12}$ of a transverse or shear acceleration wave:

$$\begin{aligned} G_u \equiv \rho\, U_{S,u}^2 &= b_1\frac{t_1 - t_2}{b_1 - b_2} = \frac{t_1 - t_2}{1 - \tilde{\rho}^2} \\ &= \frac{\sigma_1 - \sigma_2}{\tilde{\rho}^2 - 1} = \frac{2\tau}{\tilde{\rho}^2 - 1} \,, \end{aligned} \tag{4.3}$$

where τ is the shear stress:

$$\tau \equiv \tfrac{1}{2}(\sigma_1 - \sigma_2) = \tfrac{1}{2}(t_2 - t_1) \,. \tag{4.4}$$

The Lagrangian wave speeds are $\tilde{\rho}\, U_{L,u}$ and $\tilde{\rho}\, U_{S,u}$. If $\tilde{\rho} > 1$ the material is in compression, and (4.3) implies that a shear acceleration wave can propagate only if $\sigma_1 > \sigma_2$. When $\tilde{\rho} = 1$, the results of the previous section apply, and we have $G_u|_0 = G_0$ and $L_u|_0 = L_0$.

From (4.3) we have the following fundamental formula for the shear stress in uniaxial strain:

$$\tau = \tfrac{1}{2}(\tilde{\rho}^2 - 1)G_u\,. \tag{4.5}$$

Since $t_2 = t_3$, (2.3) and (4.4) imply the following well-known relation between the (compressive) longitudinal stress σ_1, the shear stress τ, and the pressure p_u in uniaxial strain:

$$p_u = \sigma_1 - \tfrac{4}{3}\tau\,. \tag{4.6}$$

On substituting (4.5) into (4.6), we obtain the following fundamental formula for p_u:

$$p_u = \sigma_1 - \tfrac{2}{3}(\tilde{\rho}^2 - 1)G_u\,. \tag{4.7}$$

We define the bulk modulus K_u in uniaxial strain by

$$K_u \equiv \rho\frac{dp_u}{d\rho} = \tilde{\rho}\frac{dp_u}{d\tilde{\rho}} = L_u\frac{dp_u}{d\sigma_1}\,. \tag{4.8}$$

Then from (4.7) and (4.2), we obtain

$$\begin{aligned} K_u &= L_u - \tfrac{4}{3}\tilde{\rho}^2 G_u - \tfrac{2}{3}(\tilde{\rho}^2 - 1)\tilde{\rho}\frac{dG_u}{d\tilde{\rho}} \\ &= (L_u - \tfrac{4}{3}G_u) - \tfrac{2}{3}(\tilde{\rho}^2 - 1)H_u\,, \end{aligned} \tag{4.9}$$

$$H_u = 2G_u + \tilde{\rho}\frac{dG_u}{d\tilde{\rho}} = \frac{1}{\tilde{\rho}}\frac{d}{d\tilde{\rho}}(\tilde{\rho}^2 G_u)\,. \tag{4.10}$$

At $\tilde{\rho} = 1$, (4.9) reduces to $K_u|_0 = L_0 - \frac{4}{3}G_0 = K_0$. From (4.9) and (4.10) it follows that $K_u = L_u - \frac{4}{3}G_u$ for all $\tilde{\rho}$ iff $H_u = 0$ iff $G_u = G_0/\tilde{\rho}^2$, but there is no reason to expect such dependence in general, and thus no reason to expect that $K_u = L_u - \frac{4}{3}G_u$ except in the limit of zero strain. Of course, by analogy with (3.5) we could have defined K_u to be $L_u - \frac{4}{3}G_u$, but then (4.8) would not hold. From (4.9) we see that for a state of compression, $K_u < L_u - \frac{4}{3}G_u$ if $H_u > 0$, and $K_u > L_u - \frac{4}{3}G_u$ if $H_u < 0$. We assume that p_u is a strictly increasing function of $\tilde{\rho}$. Then any function of $\tilde{\rho}$ may also be regarded as a function of σ_1 or p_u, and by (4.2) and (4.8) we have

$$\rho\frac{d}{d\rho} = \tilde{\rho}\frac{d}{d\tilde{\rho}} = L_u\frac{d}{d\sigma_1} = K_u\frac{d}{dp_u}\,. \tag{4.11}$$

The results up to this point are exact. We now consider some useful approximate relations. From (4.8) we have

$$p_u/K_0 \approx (\tilde{\rho} - 1) + b_0(\tilde{\rho} - 1)^2, \tag{4.12}$$

where the dimensionless constant b_0 is given by

$$b_0 = \frac{1}{2K_0}\left.\frac{d^2 p_u}{d\tilde{\rho}^2}\right|_0 = \frac{1}{2}\left(\left.\frac{dK_u}{dp_u}\right|_0 - 1\right). \tag{4.13}$$

For use in (4.12)–(4.13), note that (4.9) implies

$$\left.\frac{dK_u}{dp_u}\right|_0 = \frac{L_0}{K_0}\left(\left.\frac{dL_u}{d\sigma_1}\right|_0 - \frac{8}{3}\left.\frac{dG_u}{d\sigma_1}\right|_0\right) - \frac{8}{3}\frac{G_0}{K_0}. \tag{4.14}$$

From (3.6)–(3.7) and (4.12)–(4.13), we see that

$$p_u \approx p_h + K_0 c_0(\tilde{\rho}-1)^2\,, \tag{4.15}$$

$$\frac{p_u - p_h}{p_u} \approx \frac{p_u - p_h}{p_h} \approx c_0(\tilde{\rho}-1)\,, \tag{4.16}$$

where the dimensionless constant c_0 is given by

$$c_0 = b_0 - a_0 = \frac{1}{2}\left(\left.\frac{dK_u}{dp_u}\right|_0 - \left.\frac{dK_h}{dp_h}\right|_0\right). \tag{4.17}$$

On comparing (4.16) with equation (4.6) in Scheidler [1], we find that c_0 is also given by

$$c_0 = \frac{2}{3}\left(\left.\frac{dG_h}{dp_h}\right|_0 - \frac{G_0}{K_0}\right). \tag{4.18}$$

5 DISCUSSION

The longitudinal stress σ_1 as a function of $\tilde{\rho}$ in uniaxial strain can be obtained from normal plate impact tests. Then the relation (4.6) (which does not rely on the assumption that the response is elastic) is typically used to determine the pressure p_u in uniaxial strain given some assumptions on the shear stress τ, or to determine τ given some assumptions on p_u. It is often assumed that $p_u(\tilde{\rho})$ is equal to the pressure $p_h(\tilde{\rho})$ in a state of hydrostatic stress at density $\tilde{\rho}$ (or to some appropriate modification of p_h to include thermal effects in the shocked state). Such an approximation neglects the effects of shear strain (or shear stress) on p_u. That this effect may be significant in ceramics, geologic materials, and polymers has been emphasized by Gupta [4] and Conner [5]. These materials can sustain relatively large elastic shear strains (compared to metals), although for polymers viscoelastic effects should also be taken into account. Only elastic response is considered here. Then (4.15) implies that $p_u(\tilde{\rho})$ differs from $p_h(\tilde{\rho})$ by a term of order $(\tilde{\rho}-1)^2$ unless $c_0 = 0$, which is generally not the case. If c_0 and $p_h(\tilde{\rho})$ are known, then (4.15) provides an approximation to p_u to within an error of order $(\tilde{\rho}-1)^3$. The relative error in approximating p_u by p_h is of order $\tilde{\rho}-1$ and can be estimated by using (4.16).

In a pressure/shear (or oblique plate impact) test, a longitudinal wave propagates into the undeformed material, bringing it to a state of uniaxial strain, and a slower shear wave propagates into this uniaxially strained material. These tests yield both $\sigma_1(\tilde{\rho})$ and the shear wave speed $U_{S,u}$ (and hence G_u) as a function of $\tilde{\rho}$ or σ_1. If

the shear wave travels at the acceleration wave speed, then (4.5), (4.7), and (4.9) provide *exact* formulas for the shear stress τ, the pressure p_u, and the bulk modulus K_u in uniaxial strain as a function of $\tilde{\rho}$ or σ_1. These formulas appear to have gone unnoticed, however. Instead, it is usually assumed that $K_u \approx L_u - \frac{4}{3}G_u$. This approximate relation, together with (4.8), is then integrated to give p_u as a function of $\tilde{\rho}$; cf. Gupta [4, 6], Conner [5], and Aidun & Gupta [7]. For fused silica in the strain range $0 \leq \tilde{\rho} - 1 \leq 0.076$, the response is elastic and the shear wave speed decreases with $\tilde{\rho}$; cf. Conner [5]. In this strain range the shear wave is an acceleration wave (cf. also Abou-Sayed & Clifton [8]), so we may apply the results of §4. Using (4.9) and Conner's data, we find that at a strain of $\tilde{\rho} - 1 = 0.076$ the estimate $K_u \approx L_u - \frac{4}{3}G_u$ is low by about 29%.

Whether the shear wave in a pressure/shear test is an acceleration wave or a shock wave depends on the nonlinear elastic response of the material; cf. Davison [9]. The shear modulus G_u in §4 is defined in terms of the acceleration wave speed $U_{S,u}$, or equivalently, in terms of the speed of a plane infinitesimal sinusoidal shear wave; cf. §2. If a shear shock with speed $\bar{U}$ can propagate in the uniaxially strained material and if we set $\bar{G} \equiv \rho\bar{U}^2$, then the formulas in §4 hold approximately when G_u is replaced with $\bar{G}$. Also note that if $\bar{U} > U_{S,u}$ (as standard stability arguments would imply), then $\bar{G} > G_u$, and (4.5) and (4.7) imply that $\tau < \frac{1}{2}(\tilde{\rho}^2 - 1)\bar{G}$ and $p_u > \sigma_1 - \frac{2}{3}(\tilde{\rho}^2 - 1)\bar{G}$ in compression ($\tilde{\rho} > 1$).

We conclude with a brief discussion of thermodynamic effects, which have been neglected up to this point. If a thermoelastic material conducts heat by Fourier's law [respectively, is a nonconductor], then a longitudinal acceleration wave propagates at the isothermal [respectively, adiabatic] wave speed. However, the formula (4.3) for the speed of a shear acceleration continues to hold in either case; cf. Bowen & Wang [10]. In fact, it can be shown that (4.3) holds even if heat conduction is governed by Cattaneo's equation, which prohibits instantaneous propagation of thermal disturbances. Thus the formulas (4.5) and (4.7) for the shear stress and the pressure continue to hold. In particular, they are valid when the state of uniaxial strain has been achieved by shock loading.

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